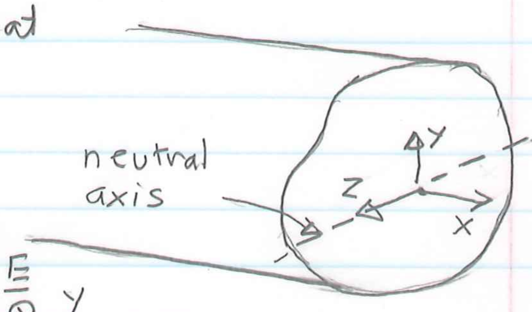
18. Bending of beams with unsymmetric cross section (linearly elastic, homogeneous, isotropic material)

The concept of neutral axis still applies. The x-y-z coordinates are positioned so that x is along the axis of the beam and z coincides with the neutral axis. Previous equations for ε_x and σ_x apply:



$$\varepsilon_x = -y/\rho \, , \quad \sigma_x = E\,\varepsilon_x = -\frac{E}{\rho}\, y$$

We want to find the conditions under which the axis of the applied moment M is parallel to the neutral axis (z axis). Since the resultant axial force on the cross-section is zero,

$$\int_{c-s} \sigma_x \, dA = 0 \Rightarrow \int_{c-s} y \, dA = 0$$

; therefore, z is a centroidal axis.

Since the resultant moment about the y axis is zero,

$$\int_{c-s} \sigma_x \cdot z \, dA = 0 \Rightarrow \int_{c-s} yz \, dA = 0$$

; therefore, if y is a centroidal axis (the origin of x-y-z can always be oriented along z to be at the centroid), then y and z are also principal axes. Thus, a moment applied to a beam whose axis is parallel to a principal axis of the cross-section will cause that principal axis to be the neutral axis. For the neutral axis at the z axis, we have

$$M = -\int_{c-s} \sigma_x \, y \, dA = \frac{EI}{\rho}$$

where M and I are about the z axis. Substituting for ρ gives

$$\sigma_x = -\frac{My}{I}.$$

All of these equations are the same as those for a symmetric cross-section.

If M is not applied about z, then we need to consider the components M_y and M_z, where y and z are principal axes of the cross-section. The stress is given by

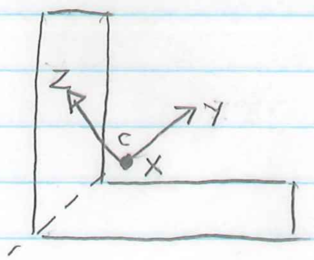
$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y x}{I_y}$$

where

$$I_z = \int_{C-S} y^2 dA \quad \text{and} \quad I_y = \int_{C-S} z^2 dA$$

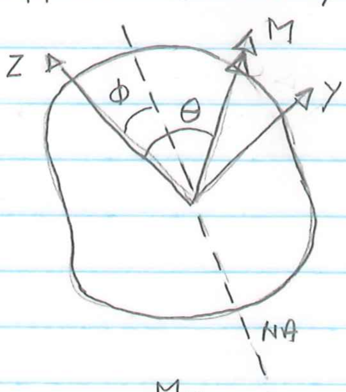
The principal axes of a cross-section can be determined as follows. Locate the centroid of the cross-section, which serves as the origin for y and z. Then orient y and z so that $\int_{C-S} yz\,dA$ equals zero. Any line of symmetry of a cross-section is a principal axis, and the other principal axis is perpendicular to it.

Example.



Cross-section is symmetric about y. Origin of x-y-z is at the centroid. y and z are principal axes.

How do we locate the neutral axis when M_y and M_z are applied (take y and z as principal axes)? In the figure,



the neutral axis makes an angle ϕ with the z axis, and the moment vector makes an angle θ with the z axis. θ is given; want to find ϕ. Along the neutral axis,

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y x}{I_y} = 0$$

This can be written as $\frac{y}{z} = \frac{M_y}{M_z}\frac{I_z}{I_y}$

Since $\frac{M_y}{M_z} = \tan\theta$ and $\frac{y}{z} = \tan\phi$,

$$\tan\phi = \frac{M_y}{M_z}\cdot\tan\theta,$$

from which ϕ can be determined.